Unified Interface-Constrained Coherence: A Geometric, Information-Theoretic, and Experimental Framework

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Abstract

We present a unified theoretical framework in which physical, informational, and complex systems are described through *interfaces*—formal instruments that define equivalence classes over underlying microstates. Coherence is quantified as an information-theoretic distance to fixed-point manifolds (recursors) intrinsic to the system, and operationally as divergence of outcome distributions relative to a chosen interface. We show that interface-induced equivalence relations imply a fiber-bundle structure on state space, rendering connections and curvature mathematically necessary for consistent comparison across spacetime. Under locality and effective field theory (EFT) assumptions, the leading curvature penalty recovers Yang–Mills structure. Introducing a Fisher-information penalty at the action level yields an interface-constrained modification of Schrödinger dynamics that maps to a Lindblad decoherence channel when the interface is treated as a physical subsystem. This produces a distinctive, falsifiable scaling of interferometric visibility with apparatus resolution. We provide explicit experimental protocols, derive bounds on the phenomenological stiffness scale m^*, and generalize the formalism to constrained networks on graphs with conserved charges that bound entanglement across cuts. The framework links geometry, information, and dynamics within a single, testable program.

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1. Introduction

Modern physical theories and complex-system models often assume that measurement, observation, or instrumentation merely *reveals* pre-existing states. We adopt the alternative premise that **interfaces are physical and informational structures that participate in the dynamics** by defining operational equivalence classes over microstates. This shift yields a unifying view: geometry emerges from equivalence; coherence becomes a measurable distance to fixed-point sets; and dynamics acquire constraint-induced penalties that are experimentally testable.

We unify five layers: (i) coherence metrics for complex systems, (ii) operational coherence via quantum instruments, (iii) geometric structure from interface-relative equivalence, (iv) interface-constrained dynamics from an action principle, and (v) interferometric protocols that falsify the theory. We further generalize to constrained networks on graphs, where conserved charges bound correlations and entanglement.

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2. Interfaces, Equivalence Classes, and State Spaces

2.1 Interfaces as Instruments

Let an **interface** be a quantum instrument $\mathcal{M} = \{\mathcal{J}_y\}$ acting on a state ρ, producing outcome distributions

$$p_\rho^{\mathcal{M}}(y) = \mathrm{Tr}[\mathcal{J}_y(\rho)].$$

Two microstates ρ, ρ' are **interface-equivalent** if $p_\rho^{\mathcal{M}} = p_{\rho'}^{\mathcal{M}}$. The set of equivalence classes defines an *observable* state space

$$\mathcal{M}_{\mathrm{phys}} = \mathcal{M}_{\mathrm{micro}} / \sim_{\mathcal{M}} .$$

2.2 Recursor Manifolds

Let $\mathcal{L}_{\mathrm{int}}$ be a generator (Hamiltonian or Lindbladian) with a fixed-point set

$$R = \{\sigma : \mathcal{L}_{\mathrm{int}}(\sigma) = 0\}.$$

We call R the **recursor manifold**.

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3. Coherence as Information-Theoretic Distance

3.1 Intrinsic Coherence

Define **intrinsic coherence** as the minimum quantum relative entropy distance to the recursor manifold:

$$C_Q(\rho) = \inf_{\sigma \in R} D(\rho \| \sigma),$$

where $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log \rho - \log \sigma)]$.

3.2 Operational Coherence

Given an interface $\mathcal{M}$ and a reference distribution $p_{\mathrm{ref}}^{\mathcal{M}}$ associated with R, define

$$C_I(\rho; \mathcal{M}) = \frac{1}{1 + D_{KL}(p_\rho^{\mathcal{M}} \| p_{\mathrm{ref}}^{\mathcal{M}})}.$$

3.3 Interface Dependence (Bell Demonstration)

For a two-qubit Bell state subject to phase noise, measurement families $Z \otimes Z$ and $X \otimes X$ yield distinct coherence trajectories: the former remains blind (constant C_I), while the latter detects decoherence (monotonic decrease in C_I). This establishes **interface relativity** of operational coherence.

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4. Geometry from Interface-Relative Equivalence

4.1 Fiber-Bundle Structure

Let $\mathcal{P}$ be the microstate manifold and $\pi : \mathcal{P} \to \mathcal{B}$ the projection onto equivalence classes induced by an interface. Under smoothness and locality, $\mathcal{P}$ forms a principal fiber bundle with base $\mathcal{B}$ and fiber G, the group of interface-preserving transformations.

4.2 Necessity of a Connection

To compare representatives across spacetime, a **connection** A_μ is required. Curvature $F_{\mu\nu}$ measures obstruction to consistent lifting. Under EFT assumptions (locality, gauge invariance, second-order equations), the leading scalar penalty is

$$\mathcal{L}_{\text{YM}} \propto \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}).$$

Thus Yang–Mills structure emerges as a minimal penalty for interface-induced redundancy.

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5. Interface-Constrained Dynamics

5.1 Fisher-Information Penalty

Introduce an action-level term penalizing sharp information gradients in the interface field $n(x)$:

$$S_C = -\frac{\hbar^2}{8m^*}\int d^4x\,\frac{(\nabla n)^2}{n},$$

where m^* is a phenomenological stiffness scale.

5.2 Modified Schrödinger Dynamics

Variation yields a correction to unitary evolution. Treating the interface as a physical subsystem and tracing it out maps the correction to a Lindblad channel:

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \frac{\eta\hbar}{8m^* d^4}[x,[x,\rho]],$$

with d the interface resolution and η a geometry-dependent constant.

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6. Experimental Protocols

6.1 Talbot–Lau Interferometry

We propose two discriminating protocols:

- **Protocol A (Fixed Talbot Order):** Vary grating period d while holding flight time and path separation constant. Prediction:

$$\ln V \propto -1/d^2.$$

- **Protocol B (Fixed Path Separation):** Adjust geometry to keep Δx constant. Prediction:

$$\ln V \propto -1/d^4.$$

Environmental decoherence models do not generically produce these scalings.

6.2 Bounds on m^*

Reanalysis of existing KDTLI/LUMI datasets yields lower bounds $m^* \gtrsim 10^3\text{–}10^4\,\mathrm{amu}$. Dedicated scans can strengthen constraints by orders of magnitude.

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7. Constrained Networks on Graphs

7.1 Global State Space

Let $G = (V, E)$ with adjacency matrix A. Each node i has a constrained Hilbert space $\mathcal{H}_i^{\mathrm{phys}} = P_i \mathcal{H}_i$, $\dim = r_i$. The global space is

$$\mathcal{H}_{\mathrm{phys}} = \bigotimes_{i \in V} \mathcal{H}_i^{\mathrm{phys}}.$$

7.2 Conserved Attention Charge

Define a weighted-degree charge

$$Q = \sum_i d_i N_i, \quad d_i = \sum_j A_{ij} w_{ij},$$

with integer-spectrum N_i. Require $[H, Q] = 0$. This quantizes and conserves **generalized attention**.

7.3 Entanglement Bound Across Cuts

For any bipartition $S|\bar{S}$ and fixed charge sector $Q = q$, the Schmidt rank is bounded by

$$\mathrm{SR} \le \sum_x \min\{D_S(x), D_{\bar{S}}(q-x)\},$$

where $D_S(x)$ is the dimension of the constrained subspace on S with $Q_S = x$. Thus

$$S(\rho_S) \le \log\left(\sum_x \min\{D_S(x), D_{\bar{S}}(q-x)\}\right) \le \log\min\left(\prod_{i\in S} r_i, \prod_{j\in\bar{S}} r_j\right).$$

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8. Cross-Domain Coherence Metrics

For any system with outcome distribution $P(t)$ and reference P_C, define

$$C(t) = \frac{1}{1 + D(P(t)\|P_C) + \eta_{\mathrm{int}}}.$$

This recovers intrinsic/operational coherence as special cases and supports tiered dynamics (decoherent → transitional → stable fixed point).

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9. Discussion

The framework unifies geometry (connections/curvature), information (relative entropy/KL divergence), and dynamics (action-level penalties/Lindblad channels) under a single principle: **interfaces define equivalence, equivalence induces structure, and structure constrains evolution**. The phenomenological scale m^* is the primary open parameter. Its physical origin—QFT, gravity, or emergent information stiffness—remains an open problem.

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10. Falsifiability and Open Problems

- **Primary test:** Distinguish $1/d^2$ vs $1/d^4$ visibility scaling under controlled Talbot scans.
- **Theoretical:** Derive m^* from microphysics or bound it universally.
- **Mathematical:** Prove topology/degree-dependent bounds on correlation growth in constrained networks.

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11. Conclusion

We have presented a single, layered framework in which interfaces, information, geometry, and dynamics cohere. The theory is not closed: its strength lies in explicit assumptions, conserved structures, and laboratory-level falsifiability. If confirmed, it establishes interface geometry as an active participant in physical law; if falsified, it constrains a broad class of information-theoretic extensions to quantum dynamics.

References

(Indicative; to be completed for submission)

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